APPLICATION OF THE BOLTZMANN-EHRENFEST PRINCIPLE TO CONTAINERLESS MICROWAVE PROCESSING IN MICROGRAVITY

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The Boltzmann-Ehrenfest (B-E) principle of adiabatic invariance has been applied to a resonant microwave cavity containing a dielectric sphere. For this system, the principle states that the time averaged energy stored in the cavity is proportional to the resonant frequency, if losses can be neglected. The stored energy, U, expressed for convenience as an increase above the stored energy of the empty cavity, varies as the position of the sphere is changed. U serves effectively as a potential for the force F exerted on the sphere, so that $F = -\nabla U$. This force equation and the B-E principle lead to the prediction that the z-component of the microwave force is given by $F_z = - C (\Delta\omega/\Delta z)$ when the sphere is displaced through the distance Δz. This prediction has been tested by direct measurement of the spatial dependence of the axial force exerted on the sphere in a cylindrical cavity excited in the TE_{111} mode, and simultaneous measurement of the resonant frequency. The constant of proportionality, C, has been determined independently by auxiliary measurements involving the Q of the cavity and the power loss. Agreement found between experiment and predictions establishes the validity of applying the B-E principle to determine forces exerted on the sample contained in the cavity. In addition, the spatial variation of the resonant frequency, for a rectangular cavity containing a lossy dielectric sphere was also measured and compared to predictions of a recent microwave perturbation model. The application of the B-E principle and the frequency shift perturbation model to the development of microwave positioning devices for future containerless materials processing studies in a microgravity environment is also discussed,

INTRODUCTION

The ability to position materials using microwave forces would be extremely valuable to NASA's Microgravity Science and Applications program. A goal of this program is to develop efficient techniques for containerless processing of materials in the microgravity environment of space. Conducting research under this program, the Jet Propulsion Laboratory has demonstrated various techniques for positioning materials. Electrostatic and electromagnetic levitation techniques are actively being explored [1]. While the forces exerted by a microwave field are too weak for practical ground-based containerless processing, they offer significant advantages for positioning materials in a microgravity environment. Through proper choice of the resonant mode, microwave forces can position samples away from the cavity walls, in a stable equilibrium configuration. Unlike the electrostatic case, no act ive positioning feedback is required. An advantage of this technique not shared by acoustic positioning, is the ability to position samples in vacuum. This can be a critical environment for certain material processing experiments.

'l'he motivation was to develop a practical, simple technique for predicting the microwave positioning potential for a low loss sample, of arbitrary size and shape in a cavity also of arbit rary si ze and shape. Theoretical predictions so far have been restricted to spherical samples. It would be a tremendous advantage for the purposes of containerless microwave processing, to be able to predict the microwave force exerted on samples in cavities, both made over a wide range of sizes and geometric shapes. in applying the Boltzmann-Ehrenfest (B-E) principle of adiabatic invariance to a single, microwave resonant mode, wc arc able to make such predictions. This paper describes an experiment that tests these predict ions.

'1'1 II :ORY

When a neutral, dielectric material is introduced into an external electric field, $\mathbf{E}_o$, the material will become polarized. The resultant electric field, **E, inside** the sample will be reduced from $\mathbf{E}_o$ by the field created from the polarization charge. The polarization vector, 1', is given by:

$$\mathbf{P} = (\varepsilon - \varepsilon_o)\,\mathbf{E} \tag{1}$$

where ε is the dielectric constant of the material and ε_0 is the permittivity of free space. The change in the potential energy of the sample after being positioned in the external field is:

$$\mathbf{U(r)} = -\frac{1}{4}\int_{V_s} (\mathbf{P} \bullet \mathbf{E}_o)\, dv_s. \tag{2}$$

As the sample is moved through an inhomogeneous external field, the. potential energy difference changes as a function of position, This gradient in the potential energy difference is equal to the negat ive of the force acting on the sample.:

$$\mathbf{F(r)} = -\nabla \mathbf{U(r)}. \tag{3}$$

Except for the case of the. small sphere, for which the wavelength of radiation is much longer than the diameter of the sphere, it is extremely difficult to calculate the expression for the electric field, $\mathbf{E}$, inside the sample, Consequentl y, the force is also difficult to calculate.

Microwave forces can be calculated using simplifying assumptions. in the electrostatic approximation, the field inside a small sphere, $\mathbf{E}$, is proportional to the external field, $\mathbf{E}_o$. The time avc.rage.cl change in the potential energy of the sample is then proportional to - E_o^2, and the force will be proportional to V E_o^2. Figure 1 shows the change in the potential energy of a small sphere as a function of position for the cylindrical TM$_{111}$ mode.. The plot is for a constant θ.

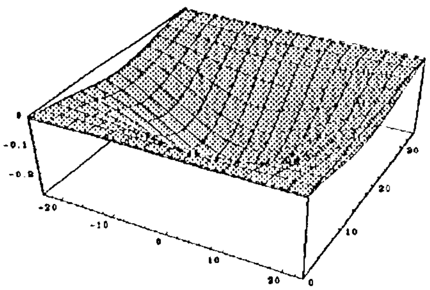

Figure 1. Potential energy difference for a small sphere in a cylindrical cavity; TM$_{111}$ mode

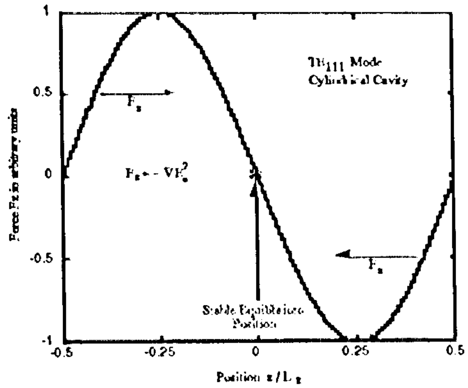

Figure 2. Microwave force exerted on a small sphere moving along the z axis at the center of a cylindrical cavity; TE$_{111}$ mode

At a constant radius, as the sample is moved along the axis of the cylinder, it will experience a force due to the interaction of the polarized dielectric with the electric field. The maximum force will occur 1/4 and 3/4 along the axis of the cavity as shown in fig. 2. At these positions, the empty cavity electric field is changing the fastest. At the center of the cavity, the electric field reaches a maximum and the microwave force exerted on the sample will be zero. The center of the cavity corresponds to a stable equilibrium position, for this mode and cavity geometry.

The B-E principle of adiabatic invariance states that for any periodic system, the ratio of the kinetic energy of the system to the frequency of the motion, is an adiabatic invariant [2]. An adiabatic invariant is a quantity that remains constant while an external constraint parameter to the system is changed, on a time scale much slower than the period of the system. This is the mechanical definition of "adiabatically". in thermodynamics, an adiabatic transformation takes place without any heat flow, dQ = O. The system is strictly conservative; all the energy in the system can be ret rieved. Any work performed on thes ystem will result in changing the internal energy of the system. The process is completel y reversible.

It should be immediately obvious that for any real, physical system, an adiabatic transformation is impossible, Work performed on a physically realizable system will always involve some. energy loss to nonconservative forces. in the case of a microwave resonant mode, strict adiabatic invariance requires that a completely lossless sample be displaced in an infinite Q cavity. The main objective of this work was to test how well the B-E principle of adiabatic invariance predicted the microwave force for a system transformed under conditions that did not strictly maintain adiabatic invariance. The B-E principle was successful] y tested for an acoustic resonant mode [3].

' in harmonic systems, such as a microwave resonant mode, where the kinetic energy is in direct proportion to the potential energy, the adiabatic invariant is simply the ratio of the total energy, U, to the frequency, ω [2]. The energy of the mode can be expressed as $U = N\hbar\omega$, where N is the number of photons in the mode and $\hbar\omega$ is the photon energ y, If U/ω is to remain constant, Nh must remain constant. The number of photons in the mode must remain the same for adiabatic invariance, to apply in this case. This constraint implies that U/ω is a constant which is just the prediction of the B-E principle,

It can be shown [4] that the change in the potential energy of the sample, $U(\mathbf{r})$, is equal to the change in the field energ y after int roducing the sample into the field, with the sources of the field held fixed. The force exerted by the microwave field on the sample, is equal to the negative change in the field energy as a function of the sample's position, Before the sample entered the cavity, the energy stored in the mode is symbolized by U_o, and the resonance frequency by ω_o. The adiabatic invariant is U_o/ω_o. For displacement of the sample in the axial direction, the force exerted on the sample will bc:

$$F_z = -\left(\frac{U_o}{\omega_o}\right)\frac{\partial\omega}{\partial z} \quad (4)$$

If we consider placing the sample at z_1 from outside. the cavity, and then position the sample at z_2 from outside the cavity, the adiabatic invariant will be the empty cavity values $(J_o/(1)0$ and the frequency shift caused by the presence of the sample, i.e.,

$$\frac{\partial\omega}{\partial z} \approx \frac{\omega_2 - \omega_1}{z_2 - z_1} \quad (5)$$

The resonance frequency and position of the sample are easily measure.cl experimentally. To determine U_o, wc utilize the definition of the quality factor, Q, of the cavity,

$$Q \equiv \frac{2\pi\,\text{Energy stored}}{\text{Energy loss /" cycle}} = \frac{\omega_o U_o}{P_\ell}, \quad (6)$$

where P_ℓ is the energy loss per second. Because the energy stored in the circuit is negligible compared to the energy stored in the cavity, the factor, QP_ℓ/ω_o, represents the stored energy in the cavity. in applying the B-E principle, it is required that the sample presence not perturb the empty cavity field distribution, the energy stored in the sample be negligible, and the energy stored in the cavity be stored in the mode. The field energy, U, can be determined from the factor, QP_ℓ/ω. The predicted B-E force can now be experimentally determined from,

$$F_z = \left(\frac{Q P_\ell}{\omega_o^2}\right) \frac{\omega_2 - \omega_1}{z_2 - z_1} \tag{7}$$

EXPERIMENTS

A. Microwave Force

The experiment was conducted by measuring the resonance frequency of the mode, the quality factor, Q, of the cavity, the power incident on and reflected from the cavity, and the axial component of the microwave force, as a function of the position of the sample. The sample position was the external constraint parameter that was changed adiabatically to test the B-E force prediction. A schematic diagram of the experimental apparatus is shown in Fig. 3.

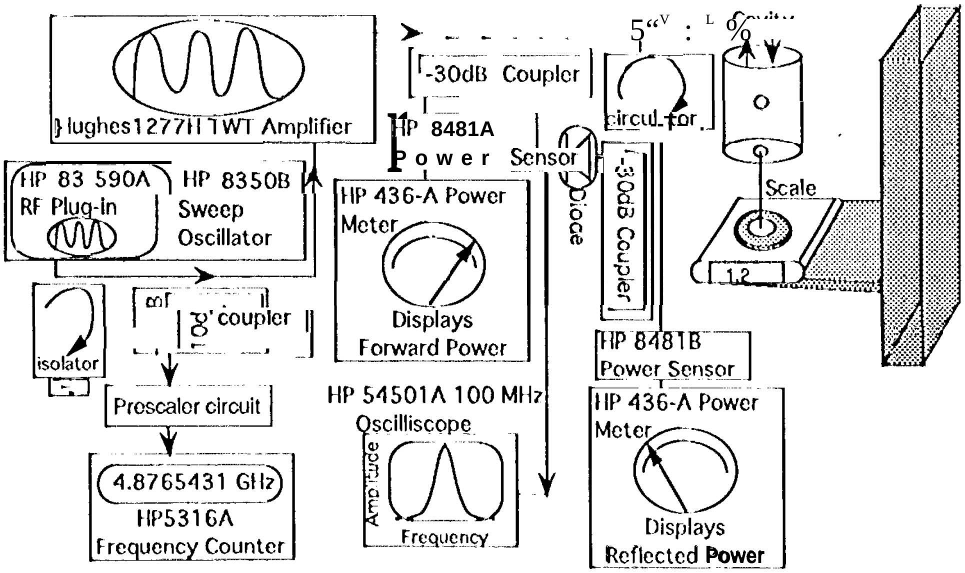

Figure 3. Experimental apparatus for determining microwave force.

An Al_2O_3 (alumina) sphere with a measured diameter of 0.95 cm. was supported cm a thin quartz rod in a cylindrical cavity that measured 4.7 cm in length and 3.42 cm in diameter. A frequency counter measured the frequenc y to an accuracy of 20kHz. The reflected power was measured directl y from the circulator using a matched 30dB attenuator to a microwave power sensor. Thirty dB of incident power was coupled out to a microwave power sensor and measured to within an accuracy of 2%. The axial component of the microwave force exerted cm the sample was measured through the quartz pedestal resting on the electronic microbalance. The accuracy of the microbalance is 0.1 mg. The microbalance was supported by a stage mounted to a dovetail slide. The microwave cavity and circuitry remained fixed, while the position of the sample changed by lowering the position of the microbalance using the slide. The relative position of the sample was measured using a vertical scale with an electronic readout, The accuracy of this measurement was 0,01 mm. Data was taken over the length of the cavity for pairs of points separated by 1.Omm, and each point within a pair, separated by 0.5 mm,

The Q of the cavity was measured at each sample position by voltage controlling the oscillator to sweep through the resonance frequency. The power reflected from the cavity through a circulator was measured using a calibrated crystal detector, The reflected power curve was fit to a theoretical Lorentzian line shape. A typical example is shown in Fig. 4. A Q = 5924 was calculated from the fit parameters. This Q measurement was reproducible to within 2%.

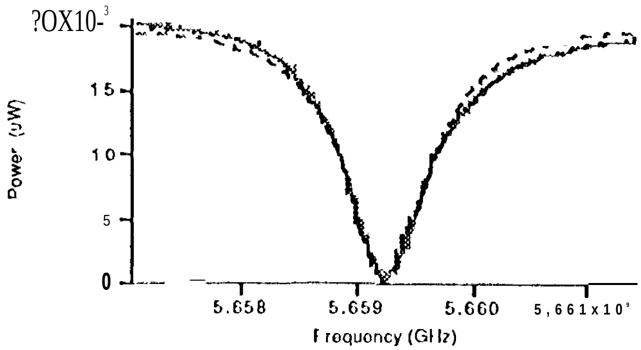

Figure 4. Typical theoretical Lorentzian line fit to reflected power sweep data. The Q of the cavity is determined from the fit parameters.

The B-E principle applied to the case of microwave resonant modes requires a single, isolated, non-degenerate, mode. While the TE_{111} mode is degenerate in the angular coordinate, this degeneracy was broken by the asymmetry of the cavity. The motivation in choosing the dimensions of the cavity as stated above, was to i solate this mode from other nearby modes. The isolation of this mode was verified by sweeping the cavity over a 500MHz bandwidth, and monitoring the mode spectrum as the sample was moved axial] y through the cavit y. The '1'111 11 mode was chosen for this experiment, because of the high measured Q value compared with the measured Q of other modes in this applicator. Maximizing the Q value used in the experiment was an important criterion, because of the limited power available out of the "l'WJ'. For this applicator, the empty cavity had a TE_{111} mode frequency $f_o = 5.72$ GHz, and a quality factor $Q_o = 5{,}780$. The maximum output of the TWT amplifier at this frequency was approximately 30 Watts. The microwaves were coupled into the cavity using an adjustable inductive loop, through the side of the cavit y body at the center 1.7+/2. The cavity was taken to be critically coupled to the external circuit when the reflected power was a minimum with respect to the frequency. The ratio of reflected to forward power ranged in value between 2% anct 7%. The cavity was critically coupled for the sample in one position, The position of the coupler was held constant there, while the data was taken as a function of sample position.

in fig. 5, the normalized measured force and the 11-1} predicted force are plotted as a function of sample position along the axis of the cavity. The measured force data represent the sample's weight change measured by the electronic microbalance. The B-ii predicted force data are calculated from the experimental parameters using eqn. 7.

Q is the measured quality factor of the loaded cavity with the sample present. P_ℓ represents the sum of the power loss in the cavity and the power loss in the circuit. Under resonant conditions these arc equal, and P_ℓ is twice the power loss measured in the cavity. The average values of Q, P_ℓ, and ω, were calculated for each pair of points, and used in the force calculation, because these values were not con st ant over the ran ge of data. The Q value was the parameter that changed the most. The highest values were measured at the top and bottom of the cavity. As the sample position moved towards the middle away from the top or bottom of the cavity, the Q value decreased by as much as 30%. This is an indication that adiabatic invariance was not strictly maintained,

This plot shows that the predicted and measured force were in good agreement over the middle half of the cavity. When the sample approached the top or bottom plate of the cavity, this size sample, with a diameter equal to 28% of the length of the cavity, interfered with the currents flowing in the walls of the cavity. This phenomenon is reflected in the Q measurement for those sample positions, It was found that the measured Q of the loaded cavity with the sample c1osc to the cavity end plates, was higher than the measured empty cavity loaded Q value, by as much as 15%. Because this Q value is used to calculate the force predicted by the, B-E principle., the predicted form was larger than the measured value,

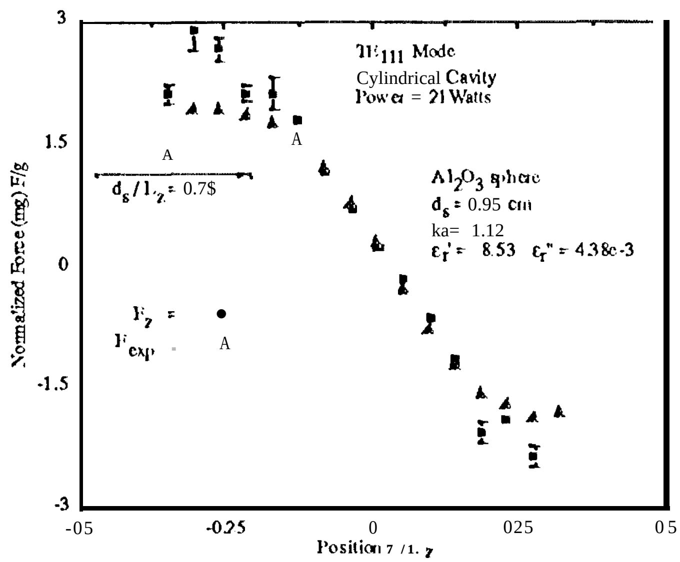

Figure 5. Comparison between measured and predicted, F_z, normalized force data vs.sample position

B. Resonant Frequency Variation

The frequency shift model predicts that the relative frequency shift, A, will be given by [5]

$$\Lambda \equiv \frac{\omega' - \omega'_o}{\omega'} = - \frac{\varepsilon_o}{4U_o}\left[\left(\varepsilon_r' - 1\right)R - \varepsilon_r'' I\right], \tag{8}$$

where R and 1 correspond to the real and imaginary components of the complex valued integral

$$-\int_{V_s} \mathbf{E} \bullet \mathbf{E}_o^* \, dv_s = R + iI. \tag{9}$$

in this integral, E represents the electric field inside the sample and $\mathbf{E}_o^*$ is the complex conjugate of the unperturbed field in the cavity. The integral is taken over the volume of the sample, 'J'he real and imaginary components have been calcul ated for a sphere in a rectangular cavit y [6].

The predictions have been experimentally tested using a 1.1 cm diameter sphere of zerodur. The TM330 mode was excited in a rectangular cavity with dimensions L_x =- 10.Z? cm., L_y = 18.1 cm., and L_z = 14.99 cm. at a frequency of 5.04 GHz. Microwaves were coupled into the cavity using a capacitive coupler located at (1/2.) L_x, (2/5) L_y L_z. A very low power level of 10 mWatts was used to ensure that the zerodur sphere did not heat, The dielectric parameters used in the theoretical fit were. $\varepsilon' = 5.48$ and $\varepsilon'' = 0.243$. Figure 6 shows that the theory very accurately predicts the relative frequency shift.

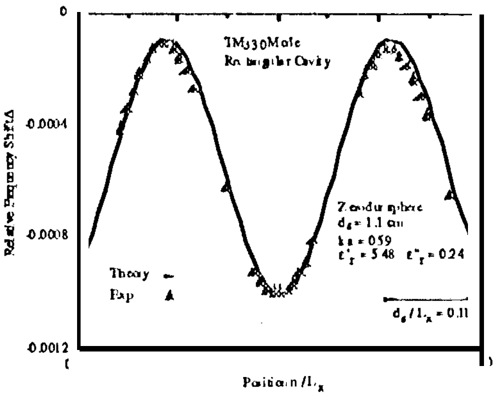

Figure 6. Comparison between experimentally measured and theoretcally predicted relative frequency shift for a sphere of zerodur, as a function of position along the x axis in a rectangular cavity.

DISCUSSION

Traditionally, theoretical calculations of the microwave force acting on a material have been restricted to the small sphere limit. The calculation required that the diameter of the sphere, "a", be much smaller than the wavelength of radiat ion. This requirement restricted the product of the wavenumber, "k", and the sphere diameter, "a", to be ka << 1. in the frequency shift experiment, ka = 0.59 and in the experimental verification of the B-E prediction, ka = 1.12. The experimental results reported here, show that the B-E principle can be used to determine the positioning potential, U(r), experienced by samples in cavities, over a wide range of sizes and geometries for each. The force predictions can be made from a straight forward measurement of the change in resonant frequency, as a function of the sample's position. Theoretical calculations for arbitrary sizes and shapes of samples and cavities would be extremely difficult.

The frequency shift model can be combined with the. B-E principle to provide an extremely powerful tool for designing microwave levitators. If it were desirable to use a spherical sample in a rectangular cavity, by coupling the predictions from these two theories, onc could predict the strength of the restoring force, the strength of the microwave torque, and the behavior of the positioning force as a function of temperature. These predictions would be valid for a broad range of sphere sizes. Because the B-E principle assumes no losses in the system, the samples would be restricted to low loss materials levitated in a high Q cavity. Our future work will include developing and testing an extended B-E theory that would account for such losses. Consequently, there would be no restrictions on the samples for the microwave force predictions.

For the purposes of NASA's Microgravity Science and Applications program, the spherical sample geometry is preferred for positioning samples that are processed in a microgravity environment. This sample geometry is desirable for microwave technology as well, when considering the spurious effects caused by the interaction of microwaves with samples containing sharp edges and corners. The rect angular cavity geometry is well suited for coupling to high power sources. The restriction of a spherical sample geometry in a rectangular cavity, is not a serious limitation to developing microwave positioners for NASA's Microgravity Science and Applications program. Eliminating the need to perform extremely difficult theoretical calculations required in determining the force acting on large size samples through the B-E prediction, is a significant achievement for the program's goals. In addition, eliminating the need to conduct frequency measurements to map out the positioning potential fox a sphere in a rectangular cavity is of great importance to this program. The design of microwave positioners can be easily optimized to ensure successful materials processing experiments conducted in a microgravity environment.

CONCLUSION

We have experimentally verified the validity of applying the B-E principle of adiabatic invariance to a single, isolated, microwave resonant mode. It was not known *a priori, how* sensitive the results would be to the restriction of adiabatic invariance. For the case of a large, low loss, spherical, sample in a cavity with a loaded Q of 5780, the B-E principle led to force predictions that closely agreed with the experimentally measured forces, over the middle half of the cavity. The B-E principle determines the positioning potential of a single microwave resonant mode, from the spatial dependence of the resonance frequency. This technique can be applied to samples and cavities of arbitrary sizes and shapes.

A recently developed frequency shift model has been verified for a large lossy sphere in a rectangular cavity. By combining this frequency shift model with the B-E principle, the positioning potential for a given mode can be predicted without experimental measurements for a large range of sphere sizes in a rectangular cavit y.

Currently, we are developing a modified B-E principle that can be applied to non conservative systems. Predictions could be made regarding the positioning potential for a lossy sample heated in a microwave resonant mode. Positioners could then be designed to incorporate both microwave heating and positioning within the same applicator. This could be easily accomplished since the stable. equilibrium position occurs at the. position of maximum electric field. For most materials, this corresponds to the position of maximum hc.sting.

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